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Particle Accelerators and Detectors

Their History, Theory, Construction, and Applications

Over time, as scientific exploits have led to greater discoveries, theoretical physics research has split towards two extremes: the extremely large and the extremely small. Research in the extremely small is almost entirely focused on particles, the smallest constituents of matter and the universe. To study such tiny, evanescent particles, however, we require the invaluable aid of particle accelerators and detectors. Particle accelerators use electromagnetic fields to drive particles to relativistic speeds, allowing researchers to analyze interactions between the most fundamental particles, study atomic structure or condensed matter sciences, and even treat cancer. Particle detectors, on the other hand, detect, trace, and identify high-energy particles within the accelerators. Particle accelerators work through electromagnetic interactions; particle detectors consist of several layers that record particles' electric, thermal, and radiation signals. In this paper, we will analyze the history, construction, properties, and of particle accelerators and detectors as well as relevant mathematical and physical concepts.

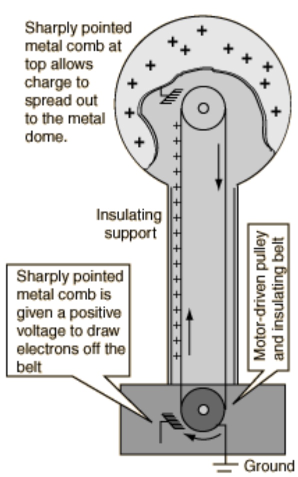
1 History

Particle accelerators have developed nearly a century of rich history. In 1927, Rolf Wideroe, a Norwegian engineer, introduced a linear accelerator, which relied on alternating currents (AC) in PhD thesis. He went on to release the Betatron principle, using a time-varying magnetic field to accelerate electrons in a circle, in a published 1928 paper.

The next particle accelerator, which relied on the principles of electrostatics to accelerate protons, was introduced in 1930 by John D. Cockcroft and Ernest T.S. Walton at the Cavendish

Laboratory in Cambridge, England[1]. Through the use of a 200-kilovolt transformer, Cockcroft and Walton performed various experiments by accelerating ions linearly through constant voltage steps with what became known as the **Cockcroft-Walton Accelerator**[2], which is currently utilized in the first step of acceleration at Fermilab. In the same year, David Sloan developed a linear accelerator which could accelerate Mercury ions at up to 1.25 MeV, while Ernest Lawrence and Milton Livingston developed the first **Cyclotron**[3]. The cyclotron encountered numerous advancements throughout the 1930, and by the 1960s, over a hundred were built in laboratories around the world.

Robert Van de Graaf introduced the **Van de Graaf Generator** in 1932. The Van de Graaf Generator could generate and maintain high voltages, consequently raising the high-energy limit. With the use of an insulating belt rather than a ladder of voltages, Van de Graaf accelerators can achieve energies of about 10 MeV.



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1 A TIMELINE OF MAJOR PARTICLE ACCELERATORS http://bt.pa.msu.edu/pub/papers/steeremsc/steeremsc.pdf
2 http://hyperphysics.phy-astr.gsu.edu/hbase/particles/accel3.html
3 http://www.accelerators-for-society.org/about-accelerators/timeliner/timeline.php

Meanwhile, in 1940, Donald Kearst applied Wideroe's Betatron principle to build the first successful **Betatron** at the University of Illinois, a circular induction accelerator used to accelerate electrons[4]. Unlike the linear induction accelerator, the betatron added magnetic bending and focusing fields in order to confine electrons to circular orbits around the isolation core.

Up to this point, accelerators were limited by maximum theoretical energy (Cockcroft-Walton and Van de Graaf by insulation breakdown due to high electric fields, cyclotrons by the increase in relativistic mass of protons and deuterons above 25 MeV, and betatrons by energy loss due to radiation by orbiting electrons). The release of the principle of **phase stability**, which established a relationship between orbiting particles and time-dependent radio frequency electric fields, by Vladimir Veksler and Edwin McMillan in 1945 (independently) led to a new path in accelerator development. After its release, phase stability was applied to a Cyclotron at the Lawrence Berkeley National Laboratory, where scientists discovered that a lower voltage could be used to produce the same acceleration.

In 1946, Frank Goward and D.E. Barnes introduced the first **Synchrotron** by modifying an old Betatron. In the same year, Luis Alvarez and Wolfgang Panofsky developed the first **Linac**. Synchrotrons and Linacs were developed throughout the 1940s and 1950s; in 1957, scientists at Dubna USSR built a Synchrotron capable of accelerating protons to 10GeV called the **Synchrophasotron**. Similar developments include the **Proton Synchroton (PS)** in 1959, which could accelerate protons to 28 GeV, and the **Alternating-Gradient Synchrotron** in 1960, which could accelerate protons to 33GeV.

By 1966, a 2 mile long accelerator had been built at the **Stanford Linear Accelerator Center (SLAC)**. Researchers at SLAC also conducted deep inelastic scattering experiments up

[4] http://web.mit.edu/22.09/ClassHandouts/Charged%20Particle%20Accel/CHAP11.PDF

to around 1968, consequently discovering a variety of quarks. Today, SLAC is helping to develop new accelerator technology for the Large Caldron Collider at CERN, and its Linac provides a unique source for X-ray laser pulses[5], which can be used to investigate extremely small and/or fast matter.

Only a year later, in 1967, scientists at Frascati, Italy built the **ADONE storage ring[6]**, which was used to explore new energy ranges in subnuclear physics, and introduced the possibility of particle-antiparticle interactions with a stationary center of mass. ADONE was used for a variety of experiments, such as the Gamma-Gamma Experiment, and produced synchrotron radiation for research in the field of solid-state physics, before being shut down in 1993 to allow for the construction of DAFNE.

In 1971, CERN scientists built the **Intersecting Storage Ring (ISR)** to collide two beams of protons, consequently giving much higher energies than collisions of single beams with fixed targets[7]. Five years later, in 1976, CERN also completed the **super proton synchrotron (SPS)**, which could accelerate protons to 400 GeV.

Scientists at DESY built the **Positron Electron Tandem Storage Ring Accelerator (PETRA)** in 1978. PETRA could collide 22 GeV electron and positron beams, and its use led to the discovery of the gluon in 1979. DESY scientists also built the first electron-proton collider, **HERA**, in 1990.

In 1983, scientists at Fermilab built the **Tevatron**, which was the first accelerator to implement superconducting magnets. From 1983 to 2011, it accelerated protons to world record energy (512 GeV) (1983), and set the world record for the number of high-energy proton-

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[5] https://www6.slac.stanford.edu/research/accelerator-research.aspx
[6] http://www.lnf.infn.it/acceleratori/adone/
[7] http://home.cern/about/accelerators/intersecting-storage-rings

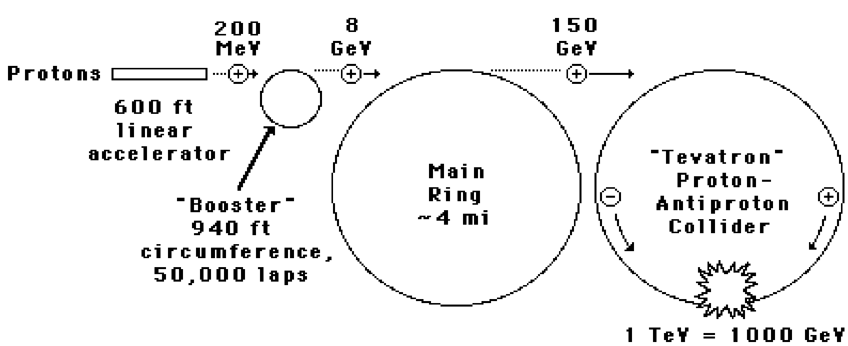
antiproton particle collisions in 1995[8]. Although it was shut down in 2011, it was the second most powerful particle accelerator in the world while in use.

But particle physics research continued. By 1989, scientists at CERN had finished building the **Large Electron Positron Collider (LEP)[9]**, which, with a 27 km circumference, was the largest electron positron collider ever built. Although it was shut down in 2000, the LEP allowed for detailed studies of electroweak interactions during its 11 years of operation.

At the turn of the century, researchers at Brookhaven National Laboratory completed the **Relativistic Heavy Ion Collider (RHIC)**, the first collider capable of accelerating ions as heavy as gold. It can accelerate heavy ions, or atoms without their outer cloud of electrons, at relativistic speeds, thousands of times per second[10].

Eight year later, in 2008, the **Large Hadron Collider (LHC)** was completed at CERN, hosted in the same 27 km tunnel as the **LEP**. Today, it is the world's largest and most powerful particle accelerator, in which particles can attain energies of up to 7000 GeV (cite)[11].

2 Application Case Study: The Tevatron[12]



http://hyperphysics.phy-astr.gsu.edu/hbase/particles/accel.html#c2

8 http://www.fnal.gov/pub/tevatron/milestones/interactive-timeline.html
9 http://timeline.web.cern.ch/timelines/The-Large-Electron-Positron-Collider
10 https://www.bnl.gov/rhic/physics.asp
11 http://cds.cern.ch/record/1165534/files/CERN-Brochure-2009-003-Eng.pdf
12 http://www.fnal.gov/pub/tevatron/tevatron-accelerator.html

2.1 Proton Preparation

The Tevatron uses a series of accelerators. Starting with hydrogen gas, a 500 ft Linac accelerates negatively charged hydrogen ions at 70% the speed of light to a carbon foil. After passing through the foil, the hydrogen ions lose their electrons, becoming positively charged protons.

Once protons have been created, the Booster, a circular accelerator, bends the Linac protons in a circular path. During each revolution, the protons accelerate due to an electric field in a radioactive cavity. By the end of the acceleration cycle, the protons have attained an energy level of up to 8 billion eV. After the Booster, the protons enter the Main Injector, in which they are injected into the Tevatron.

2.2 Antiproton Preparation

To produce antiprotons, the protons are directed towards a nickel target. The collision result in a variety of secondary particles, including antiprotons. They enter a beamline, where they are captured and focused before being injected into a storage ring, where they are accumulated and cooled. Cooling resulted in a size reduction, and an increase in brightness. After additional heating and cooling in the Recycler, the antiprotons are injected into the Tevatron.

2.3 Inside the Tevatron

Three beam lines allow the delivery of protons from the Main Injector to the neutrino targets. The beams also test detectors and carry out fixed target experiments which do not involve neutrinos. The collisions between the protons and the antiprotons are detected through the CDF and the DZero Detectors. Each detector contains many detection subsystems which identify the different types of particles emerging from the collisions. The Tevatron operated on an approximately 24 hour cycle[13].

2.4 Equations[14]

Maximum luminosity:

$$L = \frac{f_0 n_b N_p N_a}{2\pi(\sigma_p^2 + \sigma_a^2)} H(\frac{\sigma_z}{\beta^*}) = \frac{\gamma f_0 (N_p/\epsilon_{pn})(n_b N_a)}{2\pi\beta^*(1+\epsilon_{an}/\epsilon_{pn})} H\frac{\sigma_z}{\beta^*}$$

Screenshot from: http://arxiv.org/ftp/arxiv/papers/1302/1302.2587.pdf

f_0 is the Tevatron revolution frequency, n_b is the number of bunches per beam, N_p and N_a are the number of protons and antiprotons in each bunch, σ_p, σ_p are the rms transverse beam size at the interaction point (equal in the horizontal and vertical planes), γ is the relativistic beam energy, $\varepsilon_{p,a\ n}$ is the rms normalized transverse beam emittance, β^* is the optical beta function at the interaction point, σz is the rms bunch length and H is a geometrical form factor (<1).

3 Particle Accelerators

Particle accelerators have produced many recent breakthroughs in all branches of science and has opened the door for many new groundbreaking discoveries. Ongoing work at the LHC deals with the interaction between fundamental particles and its result in observable phenomena. In addition, current work at SLAC has resulted in drug development through the use of X-rays produced by particle accelerators. [15]

Although Tevatron was shut down in 2011, Fermilab has started an initiative to upgrade its current accelerators called Project X. The first stage of this plan involves upgrading existing

[13] http://arxiv.org/ftp/arxiv/papers/1302/1302.2587.pdf

[14] https://books.google.com/books?id=JW0gBAAAQBAJ&pg=PA29&lpg=PA29&dq=tevatron+physics&source=bl&ots=G3RK4J_b9t&sig=gmkwebbOnN5L49PIb2fZGllq3fM&hl=en&sa=X&ved=0ahUKEwjDpoDW5ovKAhWCLSYKHTb0A304ChDoAQgeMAE#v=onepage&q=tevatron%20physics&f=false

[15] http://www.accelerators-for-society.org/research/index.php?id=5

facilities to support current and future experiments. The second stage is the Proton Improvement Plan (PIP) and the Proton Improvement Plan 2 (PIP-II). PIP's goal is to increase the repetition of the Booster beam from 7 to 15 Hz. It will also upgrade existing infrastructure in accordance to the goals of Project X. PIP-II is aimed at upgrading hardware, increasing the power of the Main Injector to 1-2 MW and 60 GeV.[16]

With the potential discovery of the Higgs Boson in 2012, one of the most intriguing mysteries of the Standard Model may have been solved with the LHC. However, many questions still remain. The existence and origin of dark energy and dark matter, the matter and antimatter asymmetry, and the unification of the electromagnetic, strong nuclear, and weak nuclear force with gravity still remain. These big problems in physics have shaped particle accelerator development for the near future.

Modern particle detectors, as the name implies, detect particles. On spacecraft, notably the JEDI and AMS, they detect traces of radiation. The detector on JEDI was designed to detect the energies and distributions of charged particles, as well as their spectra and angular distributions in the magnetosphere of Jupiter. The detector on the AMS is designed to measure antimatter in cosmic rays and to search for evidence of dark matter. On Earth, they are used in particle colliders to find evidence of specific particles, dark energy and dark matter, and extra dimensions.

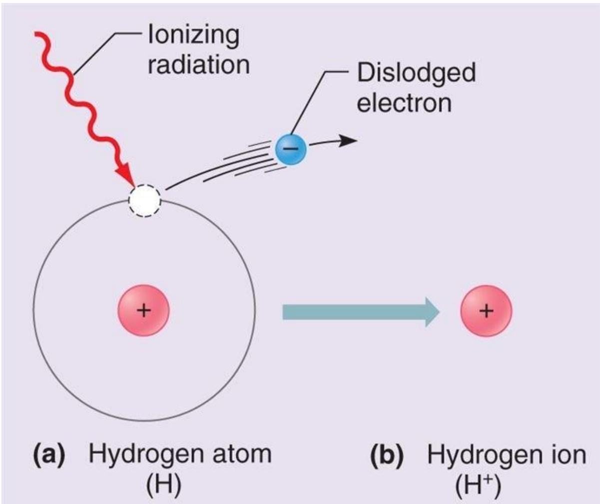
The particle accelerator is an instrument designed to move particles near the speed of light into each other, resulting in collisions that result in smaller particles and radiation. There are two types of particle accelerators: linear and circular. In linear accelerators, particles travel in a vacuum down a long tube. When the particle strikes the target at the end of the tube, particle detectors capture the event. Circular accelerators perform the same actions, except they move the

[16] http://www.fnal.gov/pub/tevatron/tevatron-accelerator.html

particles around a circular tube many times such that the particle accelerates during each pass. Once the particle speed is satisfactory, a target is placed in front of the path such that the accelerated particle can collide and particle detectors can record the event.

There are four main parts of a particle accelerator: the particle source, the tube, the klystrons, and the electromagnets. The particle source provides the particles being accelerated. The particles themselves can range from electrons to protons, while the source is different based on the particle in question. For example, if researchers wanted to examine electrons, they would use a cathode electrode to emit the electrons into the tube where they would be accelerated. There are many different types of cathode electrodes, but the most popular type involves heating a metal such that the electrons on the outermost shell of the metal atoms have enough energy to overcome the work function and break free. If the researches instead wanted a proton, they would use an ion source as the particle source. The ion source ionizes atoms, so a proton would be obtained through

$$H \quad \Rightarrow H^{+} + e^{-}$$



http://biology-forums.com/index.php?action=gallery;sa=view;id=14916

Since an ionized Hydrogen atom is just a proton.[17]

[17] http://biology-forums.com/gallery/medium_126324_22_12_13_9_17_31.jpeg

The tube is a long metal tube which has a strong vacuum inside it. These tubes are usually made up of copper, a good conductor of electricity and magnetism. These tubes are made up of many cylinders that are spaced to match the wavelengths of the microwaves generated by the klystrons. This is done so the electromagnetic waves repeat every predetermined set of cylinders. When accelerated particles bunch up at the end of one cylinder, the arrival of the electromagnetic wave pushes them to the other side.

Klystrons create the microwaves used to accelerate the particles. One end of the klystron features a cathode, which shoots out electrons usually by means of heating. An opposing electric field is then created which slows down the electrons. The change in velocity results in an acceleration which creates an electromagnetic wave, as derived from the solution to Maxwell's equations. If done correctly, the wavelength of the wave generated will lie in the microwave zone, which is desired.

The magnets used in the particle accelerators are either conventional electromagnets or superconducting magnets. Both accomplish the same task. When particles are shot out of the particle source, they do not need to move in a straight line. The magnets are placed with the same poles on opposite sides of the tube and the opposite pole directly horizontal or vertical in orientation. This creates a symmetric magnetic field that has no net force at the center. This way, if the particle is off-centered, the magnetic forces will push the particle until it is at an equilibrium in the horizontal/vertical dimension, ensuring that the particle hits the target at the correct location.

Because particle accelerators are dealing with fast charged particles and massive amounts of energy, precautions need to be taken in order to protect people. The first safety feature is the vacuum tube. The tube could be filled with common air, but then the microwaves could spark leading to damage not only to the tube, but to other objects within a close proximity as well. The air molecules could also slow down the accelerating particles, which would result in radiation from lost energy, most likely harmful gamma and X-rays.

The massive amounts of energy being used corresponds directly to a large amount of heat released. As a result cooling systems need to be in place to protect the tubes that experience the heat expulsion. Water cooling tubes around the main tube take away a lot of excess heat due to the specific heat of water. If these cooling mechanisms were not in place, then the copper tube could melt, resulting in hazardous byproducts for the operators and the surrounding structure. The vacuum seals could also break from expansion of the copper tube which would result in the aforementioned sparking and harmful radiation.

One last safety mechanism in place is the shielding in place. Because of the deceleration of electrons in the klystrons and other decelerating particles in the system, there will be a nontrivial amount of radiation released. Due to the high energy nature of the experiment, the radiation would most likely take the form of the harmful X-rays or gamma rays. In order to protect the workers in particle accelerator buildings and the surrounding citizens, the particle accelerator is surrounded by concrete and layers of dirt and Earth. This stops much of the radiation from reaching the surface. In addition, technicians are not present in the tunnels when the accelerator is in operation. This protects them from the dangerous radiation. All workers also wear radiation badges that are monitored for threatening amounts of radiation. These precautions protect the workers and surrounding communities from the harmful byproducts of particle accelerators.[18]

[18] http://science.howstuffworks.com/atom-smasher2.htm

4 Particle Detectors

In 1976, Samuel Ting, an American physicist, received the Nobel Prize in Physics for discovering the J/ψ meson. At the banquet, Ting said, "Many students...are inclined towards theoretical studies and avoid experimental work. In reality, a theory in natural science can not be without experimental foundations; physics, in particular, comes from experimental work." In this section of the paper, we will examine the math and physics concepts behind experimental constructions related to particle detectors.

Today, modern particle detectors consist of three main subsections: tracking contraptions, calorimeters, and particle-identification instruments. These three parts of the detector each reveal distinct clues about what was in the detector. Physicists can then collate their findings to examine known particles, look for new and unusual findings, and confirm or disprove current theories.

4.1 Tracking Contraptions

There are many different types of tracking detectors, the more common of which will be covered below.

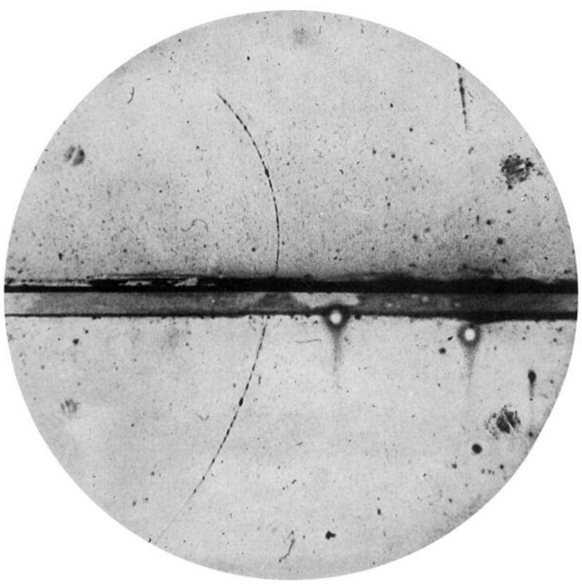
4.1.1 Cloud Chamber

A cloud chamber, as its name suggests, is a relatively small vessel (several centimeters in diameter). It is filled with air saturated with water vapor, and also has a movable piston. Additionally, there is a magnetic field surrounding the chamber. When the movable piston drops, the gas expands, and the temperature falls. As a result, the air now becomes supersaturated with water vapor. However, the extra vapor cannot condense without the presence of ions, which is where the charged particles come in.

The magnetic force formula (F= q * v x b) states that the faster something is going, the stronger the magnetic force acting on it is. Thus, the greater the energy of a particle is, the more its path will be deflected. Additionally, positively charged particles will curve in the opposite direction as that of a negatively charged one. Perhaps most importantly, the excess water vapor will condense because of the ionized particles, allowing visual photographs of the trail.

The path of a particle in a cloud chamber can be analyzed to reveal many different properties of the particle. The radius of curvature of the particle's path allows physicists to determine its velocity. Also, the thickness of the path is dependent on the particle's mass; a proton will have a thicker trail than an electron[19].

The picture on the next page is a cloud chamber photograph of the discovery of the positron in 1932. The curvature of the ion trail matched the mass to charge ratio of an electron, but it's direction indicated that the particle had a positive charge. This was the first evidence of antimatter, and won Carl David Anderson the 1936 Nobel Prize.



https://en.wikipedia.org/wiki/Positron

4.1.2 Bubble Chamber

A bubble chamber is very similar to a cloud chamber. Invented in 1952 by Donald Glaser, a bubble chamber superheats a liquid to just above its boiling point. Then, as charged

[19] http://physics.bu.edu/neppsr/2006/TALKS-2006/Tracking_Morii.pdf

particles travel through the liquid, bubbles form along its path. Just like the cloud chamber, physicists can then analyze this path.

Today, some bubble chambers are filled with liquid hydrogen, allowing physicists to study the interaction between certain particles and the hydrogen nuclei.

4.1.3 Spark Chamber

Spark chambers have two charged plates or wire grids. By keeping one side positively charged and the other negatively charged, particles that travel between the two ionize the air. Consequently, sparks jump along the ionization paths, and can be photographed, just like cloud and bubble chambers.

Today, bubble and spark chambers comprise the majority of tracking detectors used. Bubble chamber pictures have higher resolution and are more detailed than spark chamber pictures[20]. However, spark chambers can be used more selectively; physicists can set it such that only a certain particle's tracks are recorded. Thus, spark chambers are primarily used when studying rare particles.

4.1.4 Other Detectors

One obvious problem from all these different chambers is that electrically neutral particles such as neutrons and neutrinos do not interact with electromagnetic forces. For these particles, multiple different methods are used, including converting them to charged particles, nuclear reactions, and scintillation counters[21].

Another problem is particles that have an exceptionally short lifetime. The J/ψ meson mentioned earlier is one example. Its mean lifetime is $7.2*10^{-21}$ seconds[22]. As a result, it neither travels very far or interacts with matter much. The solution that physicists use today is a

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[20] http://www.phys.ufl.edu/~korytov/phz6355/note_A11_detectors.pdf
[21] http://www.kip.uni-heidelberg.de/~coulon/Lectures/Detectors/Free_PDFs/Lecture12.pdf
[22] http://www.physics.umd.edu/hep/TrackingDetectors.pdf

specialized chamber that solely tracks muons, which is usually the first layer of a particle detector.

4.1.5 Tracking Contraptions Conclusion

Today, more advanced tracking detectors are being developed that no longer require pictures. An upgraded bubble chamber, called a time projection chamber, allows physicists to track a particle three-dimensionally. One example is the CDF (Collision Detector Fermilab), which, when studying head-on collisions, can account for nearly every particle released.

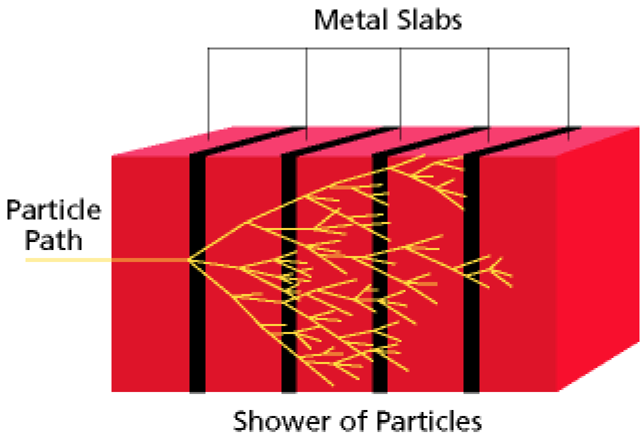
4.2 Calorimeters

Calorimeters are structures designed to “absorb”, or stop, a particle traveling through it. By doing so, a particle’s total energy can be determined. Unlike tracking contraptions, which do not disturb the charged particles themselves, calorimeters interact with particles. One advantage of calorimeters is that they can determine both charged and neutral particles’ energy. Additionally, because many particles measured by detectors are short-lived, a calorimeter can also be used to trap unwanted decayed particles.

Today, calorimeters consist of multiple layers of absorbing material. Some layer, usually called absorbers, cause the particle to break up into many particles with smaller energies. This is often called a particle shower. Other layers, called scintillators[23], are interleaved with the absorbers. When particles hit the scintillators, a fraction of their energy is converted into some measurable quantity (light or electric current). The particle shower process continues until all the particles are low energy. Then, these particles are absorbed and stop traveling. By knowing the exact fractions of energy lost in the absorbers (due to heat) and energy converted in the

[23] https://portal.uni-freiburg.de/jakobs/dateien/vorlesungsdateien/wpf2hadroncollider/kap2b

scintillators, physicists are able to determine the total energy of the original particle[24]. The picture below is a general diagram of a calorimeter.



http://ed.fnal.gov/projects/labyrinth/games/ghostbustin/calorimeter/intro4.html?name=

There are two types of calorimeters: electromagnetic and hadronic. Electromagnetic calorimeters deal with particles that interact via the electromagnetic force, such as electrons. In electromagnetic particle showers, electrons collide and produce photons. These photons undergo pair-production, where it becomes an electron-positron pair. After multiple showers, there are billions of particles, all of which have low enough energy to be absorbed by the calorimeter. Hadronic calorimeters measure the energy of particles that interact via the strong nuclear force, such as protons and neutrons.

4.3 Particle Identification

While all particles in a particle detector travel through a tracking contraption and then a calorimeter, various particles have specific interactions that allow physicists to have a general idea of what the particle may be. For example, photons show no ionization tracks in a bubble chamber, and they show dense and short electromagnetic showers during the calorimeter step. Afterwards, various methods are used to identify the exact identity. The most common methods

[24] http://www.quantumdiaries.org/2012/07/16/how-a-calorimeter-works-part-1/

used today all use the known momentum of the particle (calculated from the tracking contraption), and then calculate a particle's velocity, and thus the particle's mass.

Time of flight detectors measure the time it takes for a particle to fly a certain distance. and then divide. While seemingly simple, particles must have a low Lorentz factor (gamma value) for this method to work. Special ionization detectors measure the density of ionization along a particle's path to determine its velocity. This works well for small velocities, where the density of ionization is strongly correlated with velocity.

Additionally, in a medium with index of refraction *n*, the phase velocity of light is *c/n*. However, particle accelerators can accelerate particles in these mediums to speeds faster than *c/n* (although still less than *c*). When this occurs, the medium produces a faint radiation.

Finally, in mediums that aren't vacuums, particle can be accelerated to speeds that are faster than the speed of light in that medium. For example, in water, light travels at about 0.75*c*. When a particle travels faster than this speed, it will excite the water molecules, which will in turn emit photons with a blue light. By constructive interference, a blue glow will be formed and will propagate forward in a cone. This phenomenon was discovered by and named after Soviet physicist Pavel Cherenkov in 1934, In 1958, Cherenkov was awarded the Nobel Prize in Physics for discovering this, as well as its explanation. In particle detectors, if a particle travels faster than light would in a specific medium, it will emit Cherenkov radiation at an angle dependent to its velocity. This angle can then be used for particle identification[25].

4.4 Conclusion

Tracking contraptions, calorimeters, particle identification detectors. These three separate devices work together in harmony. Only a particle detector, master of identifying particles, needs them all. With the enormous amount of data, scientists need advanced computers and techniques

[25] http://math.ucr.edu/home/baez/physics/Relativity/SpeedOfLight/cherenkov.html

to analyze it all. However, with known data analysis methods, scientists can compare results with known theoretical predictions, discover new particles, stumble upon new phenomena, and ultimately, improve this world[26].

5 Maths and Physics

In this section we survey and explore fundamental physical concepts related to particle accelerators and detectors.

5.1 Maxwell's Equations

Maxwell's equations are a set of four equations that elegantly describe the fundamentals of electricity and magnetism, the concepts which form a basis for the physics behind particle accelerators and detectors. By controlling electric and magnetic fields, Given below are the differential forms of Maxwell's Equations in the presence of magnetic or polarizable media, the most useful form for subsequent discussions and derivations.

Gauss' Law for Electricity	$\nabla \cdot D = \rho$ $D = \varepsilon_0 E + P$ *General case* $D = \varepsilon_0 E$ *Free space* $D = \varepsilon E$ *Isotropic linear dielectric*
Gauss' Law for Magnetism	$\nabla \cdot B = 0$
Faraday's Law of Induction	$\nabla x\, E = -\frac{\partial B}{\partial t}$

[26] http://home.cern/about/how-detector-works

Ampere's Law	$\nabla\ x\ H = J + \frac{\partial D}{\partial t}$ $B = \mu_0(H+M)$ *General case* $B = \mu_0 H$ *Free space* $B = \mu H$ *Isotropic linear magnetic medium*

(Note: equations copied directly from hyperphysics: http://hyperphysics.phy-astr.gsu.edu/hbase/electric/maxeq.html)

Where E is the electric field, B is the magnetic field, D represents the electric displacement, H is the magnetic field strength, ρ is charge density, J is current density, i is electric current, M is magnetization, P is polarization, ε_0 is the permittivity constant of free space $10^7/(4\pi c^2)$ C / Vm, μ_0 is the permeability constant of free space $4\pi \times 10^{-7}$ V s / A m, and c is the speed of light. ∇· and ∇ × represent the vector operations div and curl, respectively.

Additionally, the constants ε_0, μ_0, and c have the property $\varepsilon_0 \mu_0 c^2 = 1$. The given formulation conforms to SI Units.

5.2 The Lorentz Force[27]

The Lorentz Force describes the influence of electromagnetic fields on a charged particle and its trajectory. In particular, the Lorentz force is defined as $\mathbf{F}_L = q\,\mathbf{E} + q\,(\mathbf{v} \times \mathbf{B})$, and is invariant under coordinate transformations, a property particle acceleration physics greatly depends on.

Particle beam optics or beam dynamics cover the principles behind guiding particles through electric or magnetic fields. Controlling amplitudes and locations of specific electric and magnetic fields allows for the prediction of a particle's trajectory. Thus, particles may be focused and deflected as well as accelerated as desired through carefully calculated and generated electric and magnetic fields.

[27] http://ph381.edu.physics.uoc.gr/Particle_Accelerator_Physics.pdf

It is interesting to note, by inspecting the above equation, that when considering exclusively electric fields as compared to exclusively magnetic fields, the same force may be obtained when $E = vB$ and the orientation of the magnetic field is normal to that of the particle's velocity; as a result, when the velocity of the particle is very high, i.e. when the particle is travelling at relativistic speeds and $v \approx c$, it is far more advantageous to utilize a magnetic field as opposed to an electric field to focus or deflect, since the necessary strength of an equivalent electric field would be greater by a factor of 10^8 than that of a magnetic field.

In terms of momentum and kinetic energy with regard to the Lorentz force, classical mechanics definitions lead to $\int \Delta p = \int F_L\, dt$ and $\int \Delta E_{kin} = \int F_L\, ds$. By the definition of the Lorentz force based on electric and magnetic fields, the kinetic energy reduces to $\int F_L\, ds = q \int [E + (v \times B)]\, ds = q \int E\, ds + q \int (v \times B)\, v\, dt$, where the latter term reduces to zero as a result of the dot product of velocity with a perpendicular vector (the cross product of v and B is by definition also perpendicular to v). It follows that because the kinetic energy of a particle based on its Lorentz force is dependent on the electric field but not the magnetic, that the magnetic field cannot serve to accelerate a particle. Rather, the magnetic field focuses and deflects a particle, changing the direction of the momentum vector instead of the magnitude.

5.3 Special Relativity[28]

Special relativity results from the observable phenomenon the speed of light is the same in all reference frames, much unlike relative motion in classical mechanics. Consequently, new transformation laws can be derived for a system L***** traveling with velocity $v_z = c\beta_z$ along the z-axis with respect to a stationary system L, $x = x^*$, $y = y^*$, $z = \gamma (z^* + \beta_z ct^*)$, $ct = \gamma (\beta_z z^* + ct^*)$. The relativistic factor $\gamma = 1/ \sqrt{(1 - \beta^2)}$, and all variables designated with a * are measured in L*. In matrix form,

[28] http://ph381.edu.physics.uoc.gr/Particle_Accelerator_Physics.pdf

$$\begin{pmatrix} x \\ y \\ z \\ ct \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \gamma & +\beta\gamma \\ 0 & 0 & +\beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} x^* \\ y^* \\ z^* \\ ct^* \end{pmatrix} = \mathcal{M}_{\mathrm{L}} \begin{pmatrix} x^* \\ y^* \\ z^* \\ ct^* \end{pmatrix} .$$

http://ph381.edu.physics.uoc.gr/Particle_Accelerator_Physics.pdf

The electromagnetic field transform is also given by a similar transformation matrix below, as the expansion velocity of electromagnetic waves is independent of all reference systems.

$$\begin{pmatrix} E_x \\ E_y \\ E_z \\ cB_x \\ cB_y \\ cB_z \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & 0 & +\gamma\beta_z & 0 \\ 0 & \gamma & 0 & -\gamma\beta_z & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -\gamma\beta_z & 0 & \gamma & 0 & 0 \\ +\gamma\beta_z & 0 & 0 & 0 & \gamma & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} E_x^* \\ E_y^* \\ E_z^* \\ cB_x^* \\ cB_y^* \\ cB_z^* \end{pmatrix} .$$

http://ph381.edu.physics.uoc.gr/Particle_Accelerator_Physics.pdf

A commonly observed result of special relativity is length contraction and time dilation: a rod at rest appears shorter to an observer in a moving relativistic reference frame, and vice versa, while time appears to pass slower in a moving relativistic reference frame for a stationary observer, and vice versa. A useful consequence of time dilation especially is that unstable particles moving at relativistic speeds in particle accelerators can last longer in the accelerator because the rate at which they decay in their own reference frame is much slower in the reference frame of the laboratory.

5.4 Putting Everything Together

The aforementioned physical concepts, among others, provide guidance to design devices that generate appropriate fields. In particular, multipole fields may be generated by iron dominated magnets or by skillfully placing electrical current-carrying conductors. The latter is useful in high field superconducting magnets. In the former, the shape of iron surfaces determines fields.

Because particle beams, like light rays, tend to naturally diverge, it becomes necessary to focus them at crucial points in order to study physical properties and phenomena. As it turns out, azimuthal (East-West) magnetic fields can perform on particle beams similar functions to that of optical lens on light rays, allowing a properly designed accelerator to focus the particle beams at necessary places. Thus from advanced mathematics and physics is a particle accelerator born.